Village Infants School

Mathematics, Written and Manipulatives Policy

This Policy was Agreed by the Governing Board on	4 Feb 16
The next Policy Review	4 Feb 17

Village Infants School

MATHEMATICS - WRITTEN and MANIPULATIVES POLICY

This policy should be read alongside the Mathematics Policy and Mental Calculation Policy. It contains the key pencil and paper procedures that are to be taught throughout the school. It has been written to ensure consistency and progression. Although the focus of this policy is on pencil and paper procedures and the use of manipulatives it is important to recognise that the ability to calculate mentally lies at the heart of mastery. November 2015.

Document Adopted By Governing Body	
Date:	
Signed (Chair):	
Date:	
Print Name:	
Date of Next Review:	

Aims

- To ensure a consistent and progressive approach exists within the school to secure good to outstanding progress in written calculations and use of manipulative.

- To ensure that mental calculation is not at the exclusion of written recording and use of physical apparatus. Representative strategies and mental calculation are complementary strategies as in all methods there is an element of mental processing.

- For pupils to be able to use written recordings and manipulatives to clarify their thinking and support/extend the development of more fluent and sophisticated mental strategies.

- For pupils to select and utilise methods of written calculation and manipulatives independently and be given the freedom and encouragement to develop their own methods. Although, each method will be taught pupils should not be discouraged from using previously taught methods with which they are secure, while the new concepts are becoming embedded. In addition if pupils are secure in one form of calculation differentiation should provide pupils the opportunity to progress to a more sophisticated form.

- For pupils to reflect upon which method to use to solve a problem and ask questions such as 'Can I do this in my head?' □□'Can I do this in my head or do I need equipment to help me?' 'Do I need to use a written method?' then 'Is my answer sensible?'

- For pupils to be able to clearly explain methods of recording/representation and justify why their answers are correct using sound mathematical vocabulary. Strong speaking and listening opportunities underpin good mathematics teaching.

- To share progress in written calculations with parents so that they have the confidence and knowledge to support their pupils at home with their mathematical development

Addition

The aim is that pupils use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Pupils are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition, which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers.

By the end of Key Stage One pupils should be confident in the following Mental Strategies:

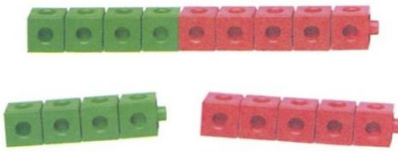
- Counting forward in ones, twos, fives and tens.
- Counting forward in steps of ones, twos, fives and tens from any multiple e.g. count in fives from 25, tens from 60.
- Knowing all doubles up to double ten.
- Knowing all the pairs of numbers that make 10 and 20.
- Adding mentally a series of one-digit numbers, (such as 5 + 8 + 4);
- Adding multiples of 10 (such as 60 + 70) or of 100, (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- Partitioning two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways. It is important that pupils's mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for addition.

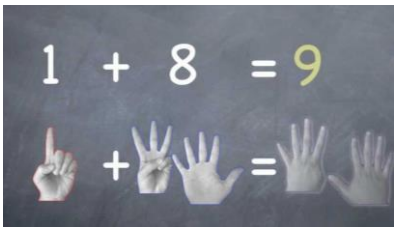
Counting in ones Starting from 0 and then from any number	Counting out loud and practicing 1:1 correspondence (knowing that each object is a separate unit) It is also important that each number represents a group of objects (e.g. 3 = 3 teddies)  

<u>Practical Addition</u> (first 'count all' and then 'count on')

Count all – 2 + 3 is counted 1, 2 and then 3, 4, 5 (out loud)

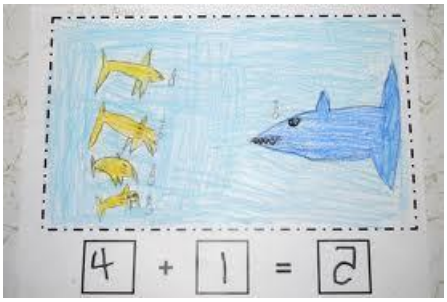
Count on – 2 + 3 is counted 2 and then 3, 4, 5

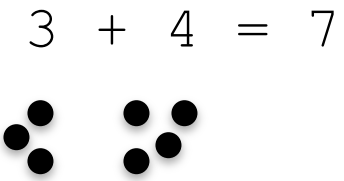




Simple addition using picture jottings

Drawing a picture
There were 4 yellow sharks and 1 blue. How many sharks were there altogether?
Dots or tally marks
3 kids were on a bus and then 4 more got on. How many were on the bus in total?

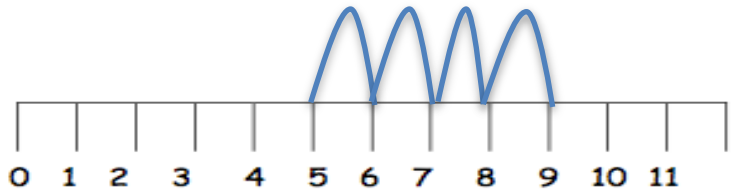




Counting in ones along a number line/track

5 pupils are at school. 4 pupils arrive late. How many pupils are at school now?

Pupils initially use a pre-drawn number line and then create their own.

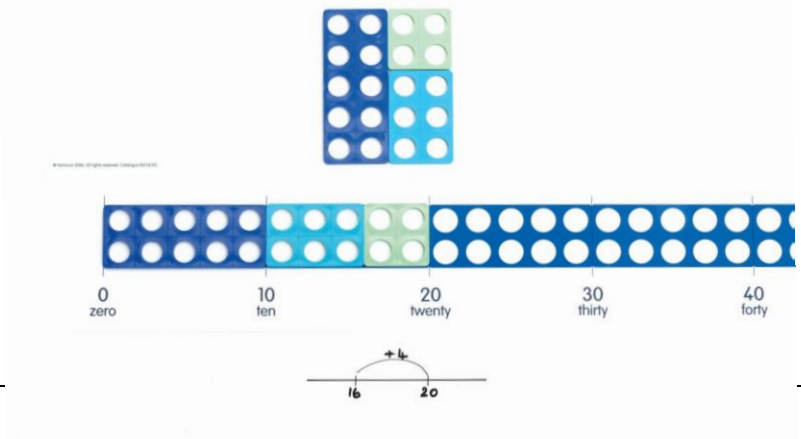


5 + 4 = 9

<u>Practical Addition</u>
Use of Numicon

To help pupils to learn number facts and visualize quantities and what digits represent.

To look for patterns and relationships in number. Use of Numicon number line to add .



Practical and informal partitioning

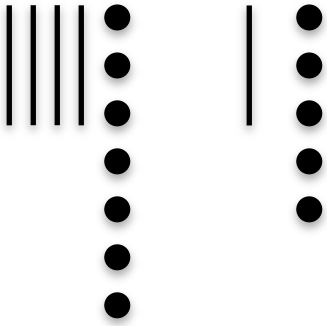
Use of practical apparatus such as Numicon and Multibase to partition and present place value of digits.

Place value cards and counters can be used to support understanding of place value.

e.g. 47 + 12 =

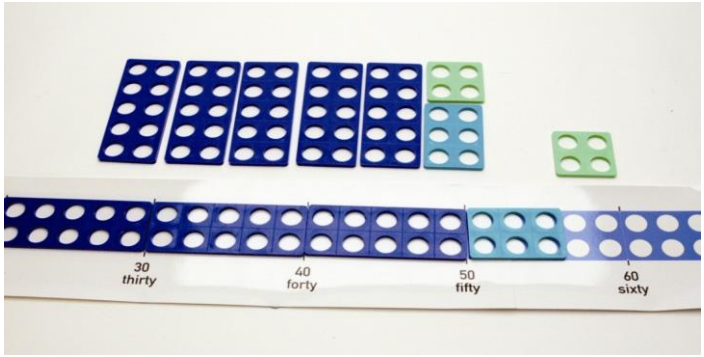
50 + 9 = 59

or 47 + 15 = 62



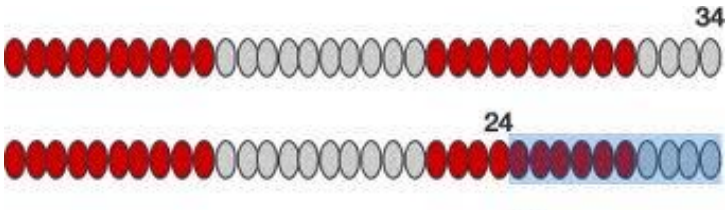
Using Addition facts to 10 to bridge the ten during addition.

Here Numicon is used to bridge the ten. E.g. in 56 + 8 the 8 is split into two 4's in order to form a number bond of 6 + 4. This enables the next multiple of 10 (60) to be reached before adding on the remaining 4.



Bead lines can also be used to show this method.

24 + 10 = 24 + 6 + 4



The Hundred Square

The Hundred Square can be used to reinforce addition in the same way as a number line where a tens boundary is not crossed. For example 24 + 5 = Also adding multiples of 10. Starting with '10 more.' Then adding 2- digit numbers that do not cross a tens boundary. For example 53 + 24 =	
The empty number line The mental methods that lead to column addition generally involve partitioning. Pupils need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps. The empty number line helps to record the steps on the way to calculating the total. One step in their develop when using a number line is to first be able to count on in tens and of course ones.	**8 + 7 = 15**  **48 + 36 = 84**  **or**  Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10
Partitioning by horizontal expansion method The next stage is to record mental methods using partitioning into tens and ones separately. Add the tens and then the ones to form partial sums and then add these partial sums. Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens.	Record steps in addition using partitioning: 47 + 76 47 + 70 = 117 117+ 6 = 123 or 47 + 76 40 + 70 = 110 7 + 6 = 13

This method builds on mental methods as each part is calculated mentally and recorded. It also makes the value of digits clear to pupils. Before calculation pupils should be able to make a sensible estimate (e.g. 487+546= is approximately 500+500=1000) so they can check the reasonableness of their answer.	110 + 13 = 123 Partitioned numbers are then written under one another, for example : 47 □□□40 □□ 7 □□76 = 70 □□□6 110 □□□13 □□123
The expanded method leads pupils to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practicing the expanded method will depend on how secure the pupils are in their recall of number facts and in their understanding of place value.	

Subtraction

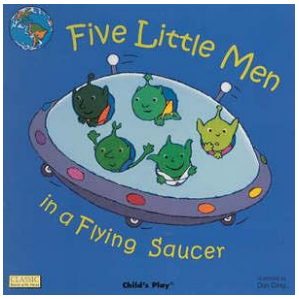
The aim is that pupils use mental methods when appropriate but, for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Pupils are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction, which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for subtraction of two-digit whole numbers. These will be introduced to pupils after they have understood the term subtraction by using objects and pictures to reinforce the idea. Pupils will always have had experience of using a numbered and empty number line as well.

By the end of Key Stage One pupils should be confident in the following Mental Strategies:

- Counting back in ones, fives and tens.
- Counting back in multiples of ones, fives and tens from any number. For example count back in fives from 35.
- Recalling all addition and subtraction facts to 10 and 20;
- Subtracting multiples of 10 (such as 160 – 70) using the related subtraction fact, 16 – 7, and their knowledge of place value;
- Partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

It is important that pupils's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

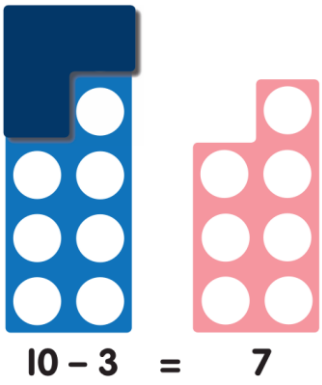
Counting backwards in ones Starting from 10 and then from any number	Counting out loud and singing number rhymes e.g. 'Five current buns' or 'ten green bottles', using visual pictures and puppets to show the process of getting less  

Practical subtraction

Practical 1:1 correspondence of finding the first number and taking away the second to find out what is left.
I found 3 pebbles on a beach but I lost one! How many did I have left?

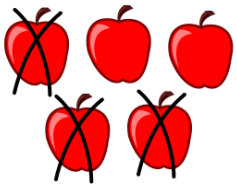
Practical subtraction

Use of Numicon to subtract using subitized pieces of equipment.



Simple subtraction using picture jottings

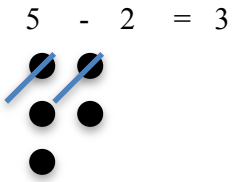
Drawing a picture
I had 5 apples but my teacher ate 3 of them. How many did I have left?



5 - 3 = 2

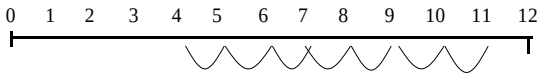
Dots or tally marks
There were 5 people of the bus but 2 got off at the first stop. How many people were still on the bus?

5-2=3



Counting back in ones along a number line/track

11 pupils are at school. 7 pupils go home because they feel sick. How many pupils are left in school? Pupils could use a pre-drawn number line and then begin to create their own.

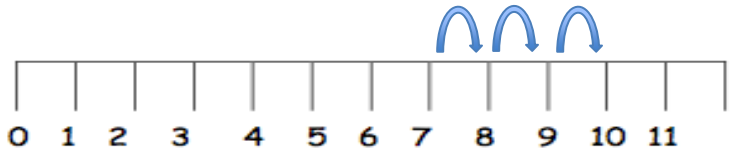


11 – 7 = 4

PUPILS SHOULD USE COUNTING BACK BRIEFLY IN THEIR DEVELOPMENT WHEN SUBTRACTING SMALL SINGLE DIGIT NUMBERS AND THEN MOVE TO USING COUNTING ON IN TERMS OF FINDING THE DIFFERENCE PREDOMINANTLY.

Finding the difference

There are 10 pupils in our class today and 7 of them are having school dinner. How many are having packed lunch.
Pupils are taught to count on from the smallest number to find the difference. (This can be done in their head or on a number line)



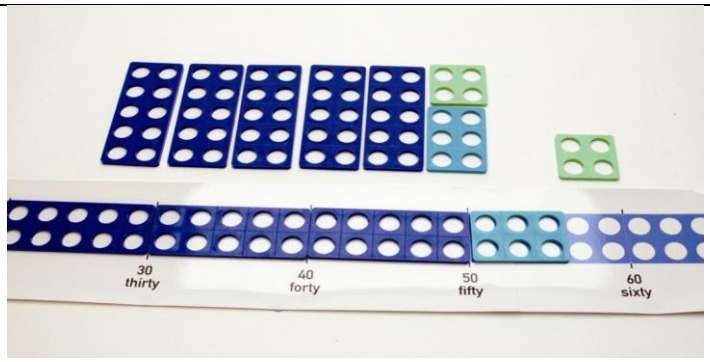
Using Addition facts to 10 to bridge the ten during subtraction by counting up to find the difference.

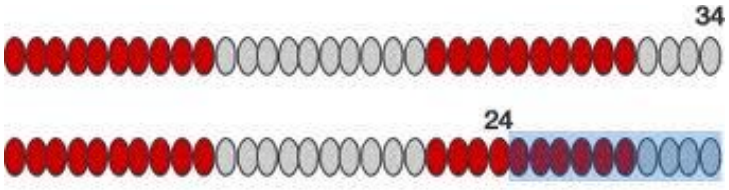
Here the use of a Numicon can be used to help bridge the ten when counting up to find the difference.
E.g. in 64 – 56 =

4 in added to the 56 to reach 60 and then 4 again to reach 64. Thereby finding the difference of 8 by counting up.

Bead lines can also be used to show this method.

E.g. 34 – 24 = 6 + 4



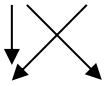


Practical and informal partitioning

Use of practical apparatus such as Multibase and Numicon.

Place value cards and counters can be used to additional support understanding of place value.

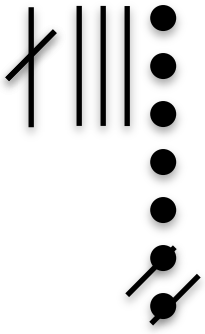
e.g. 47 - 1 2 =



30 + 5 = 35

This works when the units do not bridge the ten.

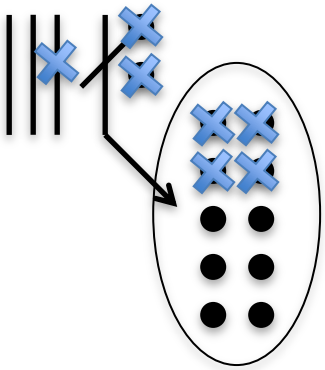
or 4 7 - 1 2 = 35



Practical partitioning where the units bridge the ten/hundreds

As there are only 2 units pupils should be taught to **exchange** 1 ten for ten units. This now means that there are 12 units and 2 tens (still 42). Pupils are then able to subtract 6 units and 1 ten, as in the previous example.

4 2 - 1 6 = 2 6



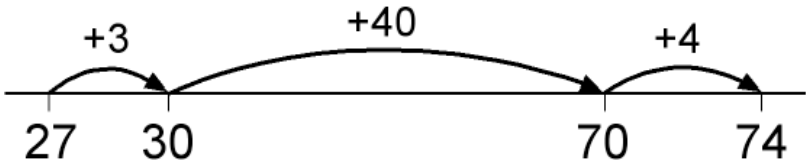
Using an empty number line to find the difference Finding an answer by counting up

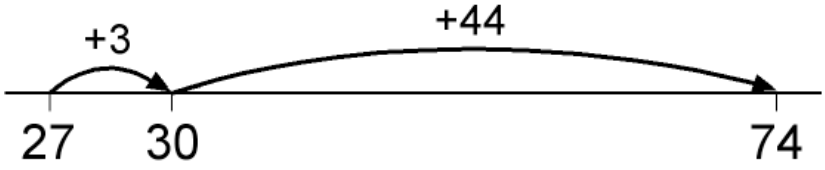
The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeepers method).

With practice, pupils will need to record less information and decide whether to count back or forward. It is useful to ask pupils whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.

74 – 27 =

or:





NOTE AT VILLAGE WE RECORD THE JUMPS WHEN SUBTRACTING BELOW THE LINE. NOT AS ILLUSTRATED.

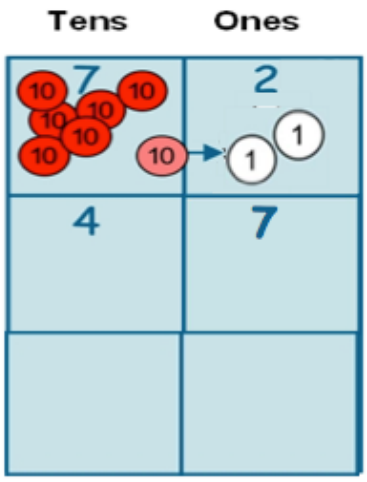
Partitioning

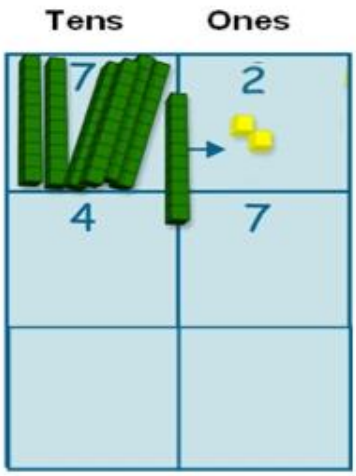
Subtraction can be recorded using partitioning to write equivalent

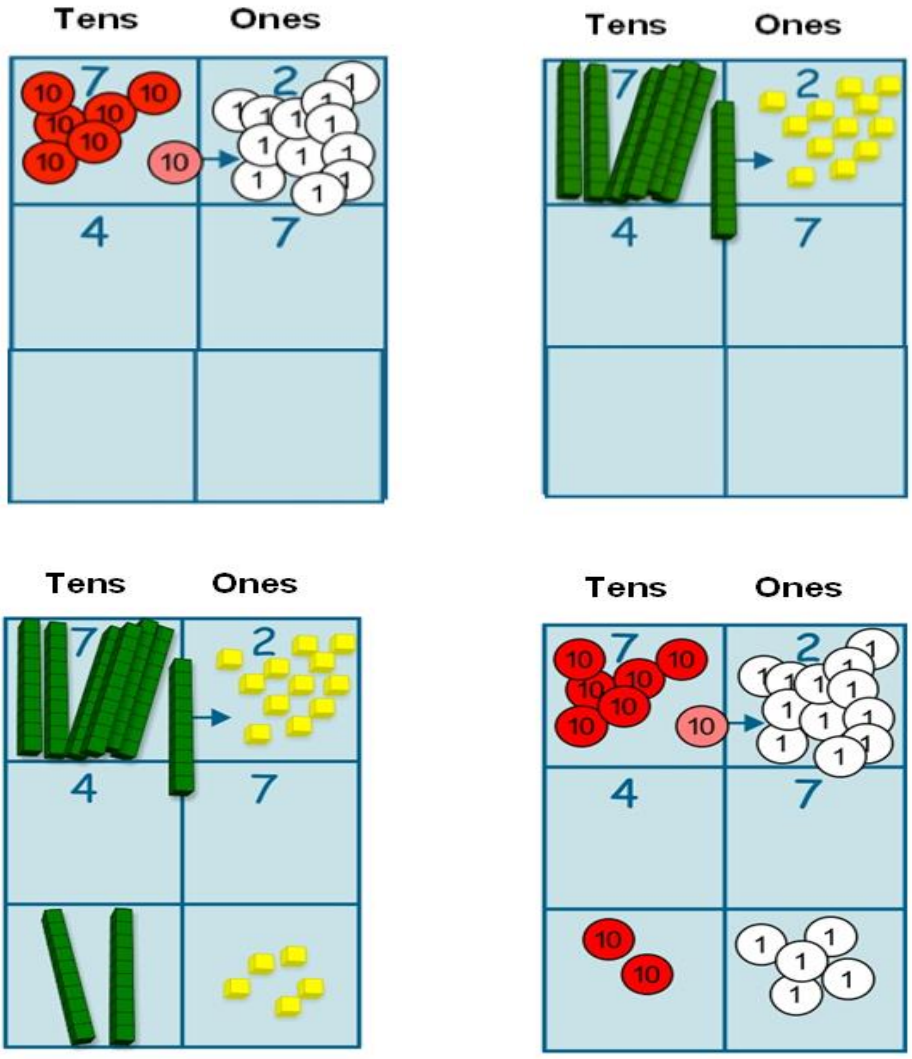
Subtraction can be recorded using partitioning:

calculations that can be carried out mentally. For 74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 7 in turn. **This use of partitioning is a useful step towards the most commonly used column method, decomposition**	74 – 27 74 – 20 = 54 54 – 7 = 47
Expanded layout, leading to column method (Decomposition) Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens. This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills. **The expanded method leads pupils to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practicing the expanded method will depend on how secure the pupils are in their recall of number facts and with partitioning.**	Example: 563 - 241, no adjustment or decomposition needed **Expanded method** 500 + 60 + 3 − 200 + 40 + 1 300 + 20 + 2 Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example: 72 - 47, **adjustment from the tens to the units** so that units can be taken away using Multibase and place value counters.







Multiplication

The aim is that pupils use mental methods when appropriate, but for calculations that they cannot do in their heads, they can use apparatus for support. Understanding is important and pupils should be able to prove their answers using a number of methods.

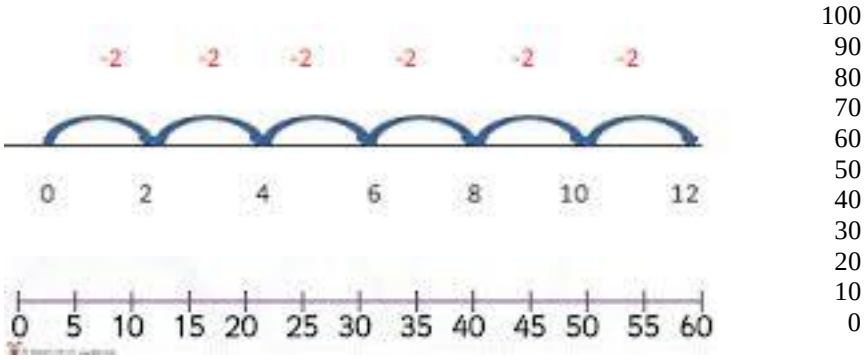
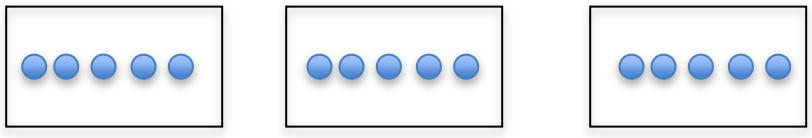
To multiply successfully, pupils need to be able to:

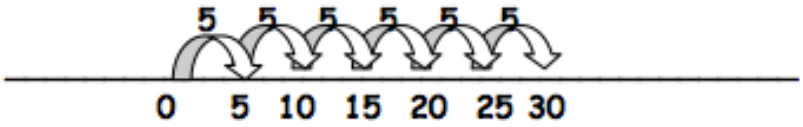
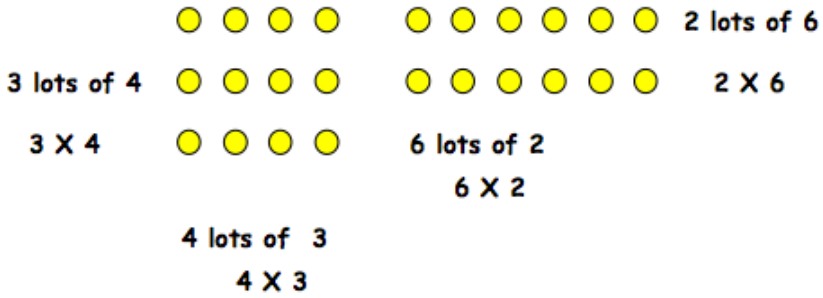
- Count in steps of different multiples, twos, fives, tens.
- Recall all multiplication facts to 10 x 10.
- Partition numbers into multiples of one hundred, ten and one.
- Apply multiplication facts e.g. 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;

- Double numbers to 10, 20, 50, 100.
- Add two or more single-digit numbers mentally;
- Add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;

It is important that pupils's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Develop the mental image of multiplication

Putting objects into equal groups Putting objects into equal groups and then checking there are for example, 2 in each group. Begin counting in equal steps by counting the number in 2 groups and then 3 and then 4 etc	 2 4 6 8
Counting in equal steps, starting with 2s, 10s and 5s, then progressing to 3s, 4s etc. Using practical apparatus such as numicon.	 3 6 9 12
Counting in equal steps, starting with 2s, 10s and 5s, then progressing to 3s, 4s etc. Understanding how to count in these steps is an important foundation to learning multiplication facts (tables)	
Multiplication as repeated addition 5 x 3 = There are 5 cakes in a pack. How many cakes in 3 packs? Dots or tally marks are often drawn in groups. This shows 3 groups of 5.	 5 + 5 + 5
Number lines	6 x 5 = 30

This model illustrates how multiplication relates to repeated addition Pattern work on a 100 square helps pupils begin to recognise multiples and rules of divisibility Using Numicon number line to solve repeated addition problems by laying pieces upon track.	or 5 + 5 + 5 + 5 + 5 + 5 = 30  
Arrays Successful written methods depend on visualising multiplication as a rectangular array. It also helps pupils to understand why 3 X 4 = 4 X 3	

Division

The aim is that pupils use mental methods when appropriate but, for calculations that they cannot do in their heads, they need to be able to select and use apparatus. Understanding is important and pupils need to be able to prove their answers are correct.

To divide successfully in their heads, pupils need to be able to:

- Count backwards in different steps, twos, fives and tens first.
- Understand and use the vocabulary of division
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- Halves of numbers to 20, 50, 100.
- Recall multiplication and division facts to 10 × 10.
- Know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5.
- Understand and use multiplication and division as inverse operations.
- **U**nderstand division as repeated subtraction (Grouping)
- Estimate how many times one number divides into another – for example, how many sixes there are

in 47.

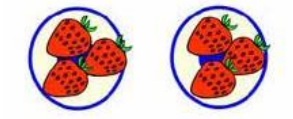
- Know subtraction facts to 20 and to use this knowledge to subtract multiples of 10 e.g.120 – 80.

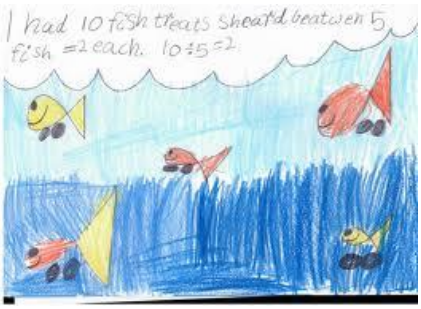
It is important that pupils's mental methods of calculation are practised and secured.

Division by sharing

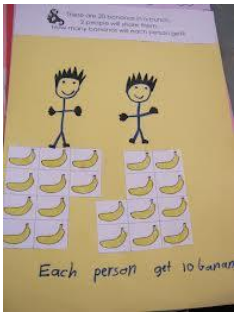
Practical sharing along with more pictures and jottings.

6 strawberries shared between 2 pupils. How many strawberries do they get each? 6÷2 =





I had 10 fish treats shared between 5 fish. 10 ÷ 5 = 2



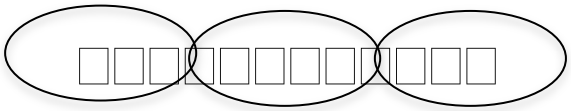
Two boys shared 20 bananas. How many did they get each? 20 ÷ 2 = 10

Sharing should only be used briefly as a precursor to grouping, which is a more preferable method and should be moved onto as soon as possible. Solving division by grouping strengthens mental calculation strategies.

Division by grouping

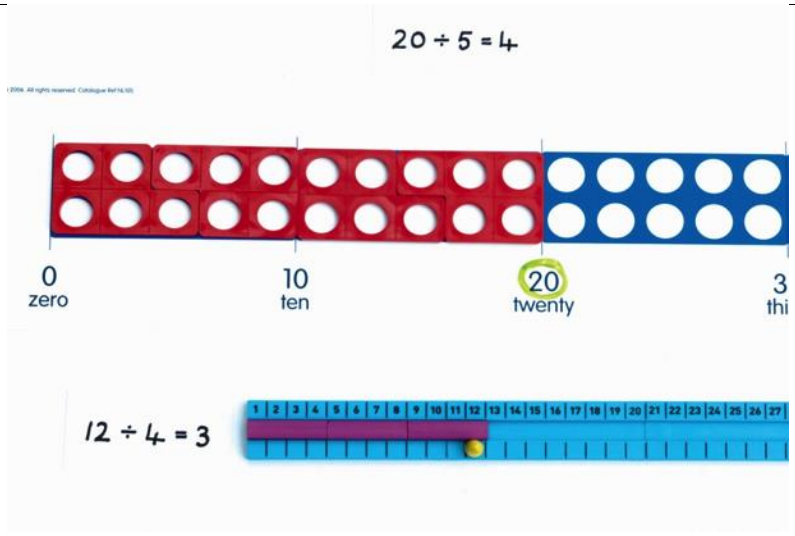
4 apples are packed in a basket. How many baskets can you fill with 12 apples? 12÷4 =

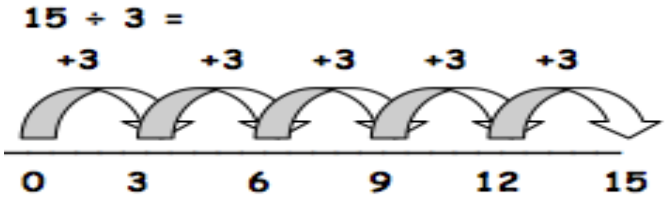
Practical grouping with 12 objects, grouped into 4's. Dots or tally's marks can be split up into groups.
E.g. draw 12 dots representing apples and grouping into 4's to find how many groups. 12÷4 =3



Numicon number line

How many 5's are in 20.



Number lines (Repeated +) Counting on in equal steps based on adding multiples up to the number to be divided Counting back in equal steps based on subtracting multiples from the number to be divided **Note Counting on is a powerful tool for mental calculation but does not lead onto written calculation for division**	 A chocolate bar costs 3p. how many can I purchase for 15p?
Finding remainders using Numicon. 37 ÷ 5 = 7 r2